Market Making Under Inventory Risk with Adverse Selection

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1. Abstract

We study optimal market making in a limit order book where the dealer faces inventory risk and adverse selection from informed traders, following the Avellaneda–Stoikov (2008) framework extended to include asymmetric information costs. The dealer controls bid and ask quote depths to maximise expected terminal wealth minus a quadratic inventory penalty, leading to a Hamilton–Jacobi–Bellman equation whose solution yields closed-form reservation prices and optimal spreads. The reservation price shifts linearly with inventory and remaining horizon, while the optimal spread decomposes into a risk-aversion component proportional to $\gamma\sigma^2\tau$ and an adverse-selection component $\frac{2}{\gamma}\ln\left(1+\frac{\gamma}{\kappa}\right)$. Simulated inventory paths under the optimal policy exhibit mean-reverting behaviour governed by the asymmetric quoting rule, with the dealer widening quotes on the side that would exacerbate a large inventory position.

2. Introduction

A market maker continuously posts limit orders on both sides of the book, earning the bid-ask spread in exchange for providing liquidity. Two fundamental risks attend this activity. **Inventory risk** arises because filled orders accumulate a position in the underlying asset, exposing the dealer to adverse mid-price movements. **Adverse selection** arises because informed traders preferentially hit the side of the market that moves against the dealer.

The seminal contribution of Avellaneda and Stoikov (2008) frames the dealer's problem as a stochastic optimal control problem on a finite horizon. The mid-price follows a Brownian motion; order arrivals are modelled as inhomogeneous Poisson processes whose intensities decay exponentially in the posted depth. The optimal policy is obtained in closed form via an exponential transformation of the HJB equation.

This note develops the model, derives the closed-form solution, and analyses the structural properties of the optimal quotes through four figures. Section 2 specifies the model. Section 3 solves the HJB equation. Section 4 presents the optimal quoting policy. Section 5 discusses adverse selection extensions. Section 6 presents numerical results. Section 7 concludes.

3. Model

3.1 Mid-Price and Wealth Dynamics

Let S_t denote the mid-price, following arithmetic Brownian motion:

$$dS_t = \sigma \, dW_t, \tag{3.1}$$

where $\sigma > 0$ is the volatility and W_t is a standard Brownian motion on $(\Omega, \mathcal{F}, \mathbb{P})$.

The dealer's cash process X_t evolves as trades arrive:

$$dX_t = \delta_t^a \, dN_t^a + \delta_t^b \, dN_t^b, \tag{3.2}$$

where $\delta_t^a, \delta_t^b > 0$ are the ask and bid depths (half-spreads above and below the reservation price), and N_t^a, N_t^b are counting processes for ask and bid executions.

The inventory satisfies:

$$dq_t = -dN_t^a + dN_t^b. \tag{3.3}$$

3.2 Order Arrival Model

Arrivals follow inhomogeneous Poisson processes with intensities:

$$\lambda_t^a = A \, e^{-\kappa \delta_t^a}, \qquad \lambda_t^b = A \, e^{-\kappa \delta_t^b}, \tag{3.4}$$

where $A > 0$ is the baseline arrival rate and $\kappa > 0$ measures the sensitivity of order flow to the posted depth. Deeper quotes attract fewer orders but earn more per execution.

3.3 Objective

The dealer maximises expected terminal wealth subject to an inventory penalty:

$$V(x, q, s, t) = \sup_{\delta^a, \delta^b} \mathbb{E}\left[X_T + q_T S_T - \frac{\gamma}{2} q_T^2 \,\middle|\, X_t = x, \, q_t = q, \, S_t = s \right], \tag{3.5}$$

where $\gamma > 0$ is the risk-aversion parameter and T is the trading horizon. The terminal term $-\frac{\gamma}{2} q_T^2$ penalises residual inventory at the close.

4. HJB Equation and Solution

4.1 HJB Equation

By the dynamic programming principle, V satisfies:

$$\partial_t V + \frac{1}{2}\sigma^2 \partial_{ss} V + \sup_{\delta^a} \{\lambda^a [V(x+\delta^a, q-1, s, t) - V]\} + \sup_{\delta^b} \left\{\lambda^b \left[V(x+\delta^b, q+1, s, t) - V\right]\right\} = 0, \tag{4.1}$$

with terminal condition $V(x,q,s,T) = x + qs - \frac{\gamma}{2}q^2$.

4.2 Exponential Transformation

Following Avellaneda and Stoikov (2008), separate V as:

$$V(x,q,s,t) = x + qs + u(q,t), \tag{4.2}$$

where u captures the inventory value. Substituting yields:

$$\partial_t u - \frac{1}{2}\gamma\sigma^2 q^2 + \sup_{\delta^a} \{\lambda^a [u(q-1,t) - u(q,t) + \delta^a]\} + \sup_{\delta^b} \left\{\lambda^b \left[u(q+1,t) - u(q,t) + \delta^b\right]\right\} = 0. \tag{4.3}$$

Define $w(q,\tau) = e^{-\gamma u(q, T-\tau)}$ with remaining time $\tau = T - t$. After substitution and the approximation $A/\kappa \gg 1$ (deep liquid book), one obtains a linear PDE in w amenable to closed-form solution.

4.3 Reservation Price

The dealer's **reservation price** — the mid-price adjusted for inventory risk — is:

$$r(s,q,\tau) = s - q\gamma\sigma^2\tau. \tag{4.4}$$

This is the certainty-equivalent value of the current position: holding inventory $q > 0$ lowers the effective mid-price by $q\gamma\sigma^2\tau$ because the dealer is exposed to adverse moves over the remaining horizon τ.

4.4 Optimal Spread

The optimal bid and ask depths around the reservation price are:

$$\delta^{a*} = \frac{\gamma\sigma^2\tau}{2} + \frac{1}{\gamma}\ln\left(1 + \frac{\gamma}{\kappa}\right), \qquad \delta^{b*} = \frac{\gamma\sigma^2\tau}{2} + \frac{1}{\gamma}\ln\left(1 + \frac{\gamma}{\kappa}\right). \tag{4.5}$$

The total optimal spread is:

$$\delta^{a*} + \delta^{b*} = \gamma\sigma^2\tau + \frac{2}{\gamma}\ln\left(1 + \frac{\gamma}{\kappa}\right). \tag{4.6}$$

The spread has two additive components: - **Inventory/risk component** $\gamma\sigma^2\tau$: grows with risk aversion, volatility, and remaining time. - **Adverse-selection component**

$\frac{2}{\gamma}\ln(1+\gamma/\kappa)$: decreases as κ grows (thinner book $\to$ tighter spread premium).

5. Optimal Quoting Policy

The dealer posts:

$$s_t^a = r_t + \delta^{a*}, \qquad s_t^b = r_t - \delta^{b*}, \tag{5.1}$$

where $r_t = S_t - q_t\gamma\sigma^2\tau$ is the reservation price. The asymmetry in quotes enters through the reservation price: a long dealer ($q > 0$) shifts both quotes down, making the ask more competitive (to sell) and the bid less competitive (to avoid buying more).

5.1 Adverse Selection Extension

In the presence of informed traders with probability α, the effective arrival intensities become:

$$\tilde{\lambda}^a = (1-\alpha)\lambda^a + \alpha \cdot \mathbf{1}[\text{ask price favourable to informed}], \tag{5.2}$$

and symmetrically for bids. The adverse-selection-adjusted spread adds a Glosten–Milgrom term:

$$\delta_{\text{AS}}^{a*} = \delta^{a*} + \frac{\alpha}{1-\alpha} \cdot \mathbb{E}[|dS|], \tag{5.3}$$

penalising each side by the expected loss to an informed counterparty. In practice this is calibrated from trade-sign imbalance regressions (Kyle, 1985; Glosten–Milgrom, 1985).

6. Algorithm

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Input: sigma, gamma, kappa, A, T, dt
Output: bid/ask quote sequence {s^a_t, s^b_t}

Initialise: q = 0, t = 0, X = 0
While t < T:
    tau = T - t
    r   = S_t - q * gamma * sigma^2 * tau
    half_spread = 0.5 * gamma * sigma^2 * tau
    as_adj      = (1/gamma) * ln(1 + gamma/kappa)
    delta_a = half_spread + as_adj
    delta_b = half_spread + as_adj
    s_ask = r + delta_a
    s_bid = r - delta_b
    Post (s_ask, s_bid) to order book
    Simulate arrivals:
        N_a ~ Poisson(A * exp(-kappa * delta_a) * dt)
        N_b ~ Poisson(A * exp(-kappa * delta_b) * dt)
    Update:
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        X = X + delta_a * N_a + delta_b * N_b
        q = q - N_a + N_b
        S_t = S_t + sigma * sqrt(dt) * Z,    Z ~ N(0,1)
    t = t + dt
Return X + q * S_T - 0.5 * gamma * q^2
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7. Numerical Results

7.1 Reservation Price

The reservation price $r - s = -q\gamma\sigma^2\tau$ is linear in both inventory q and remaining time τ.

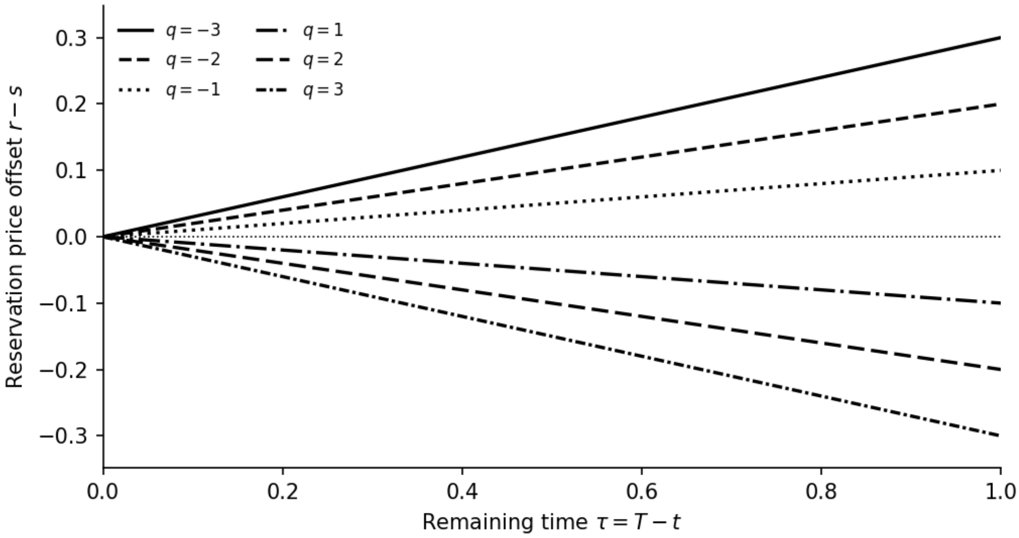

Figure 1: Reservation price offset $r - s$ as a function of remaining time τ for six inventory levels $q \in \{-3, -2, -1, 1, 2, 3\}$. Long inventories ($q > 0$) produce negative offsets; short inventories produce positive offsets. The offset magnitude grows with τ, reflecting the longer exposure horizon.

7.2 Asymmetric Quoting

When $q \neq 0$, the dealer shifts both quotes through the reservation price, creating an asymmetric execution probability.

7.3 Inventory Dynamics

Under the optimal policy, inventory is self-correcting: large positive q tightens the ask and widens the bid, increasing execution probability on the ask side.

7.4 Spread Sensitivity to Risk Aversion

The total spread $\delta^a + \delta^b$ increases with both γ and τ, but differently at short and long horizons.

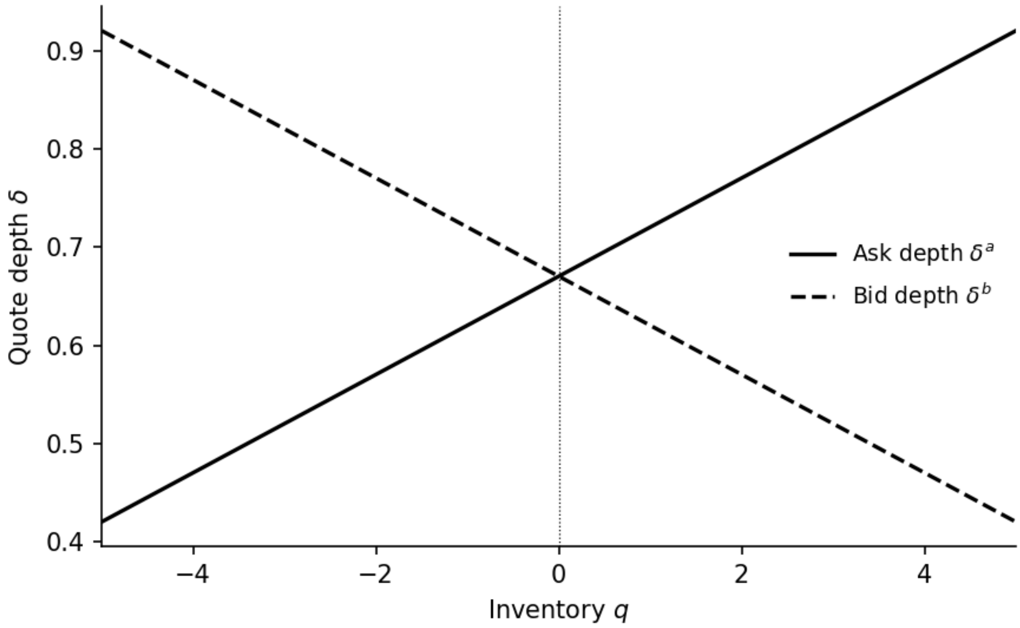

Figure 2: Optimal ask depth δ^a (solid) and bid depth δ^b (dashed) as functions of inventory q at $\tau = 0.5$. At $q > 0$ the ask depth decreases (more competitive ask to shed inventory) and the bid depth increases (less competitive bid to avoid accumulation). The curves cross at $q = 0$ where the policy is symmetric.

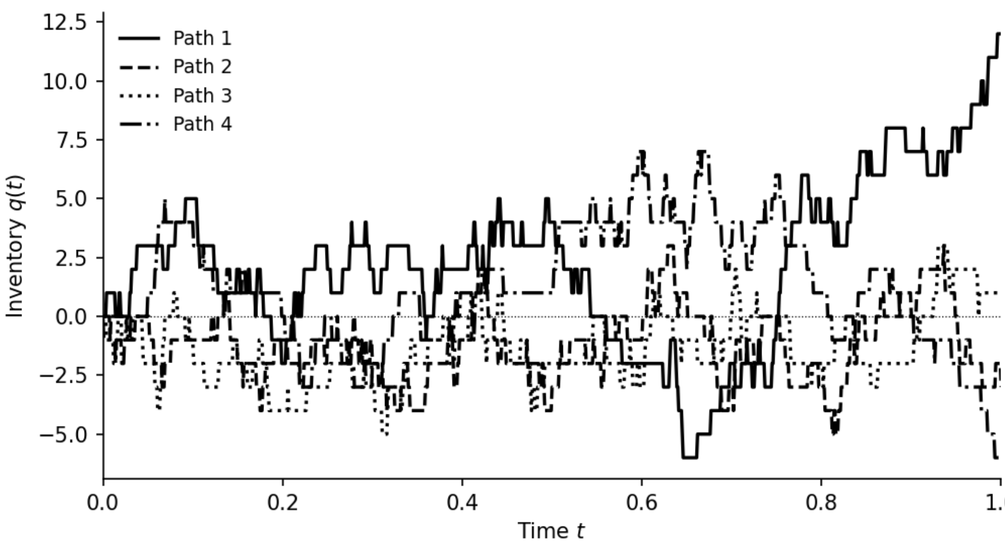

Figure 3: Four simulated inventory paths $q(t)$ under the Avellaneda–Stoikov optimal policy with parameters $\sigma = 1$, $\gamma = 0.1$, $\kappa = 1.5$, $A = 1$. The policy prevents sustained drift away from zero: whenever q grows large, quote asymmetry accelerates mean reversion.

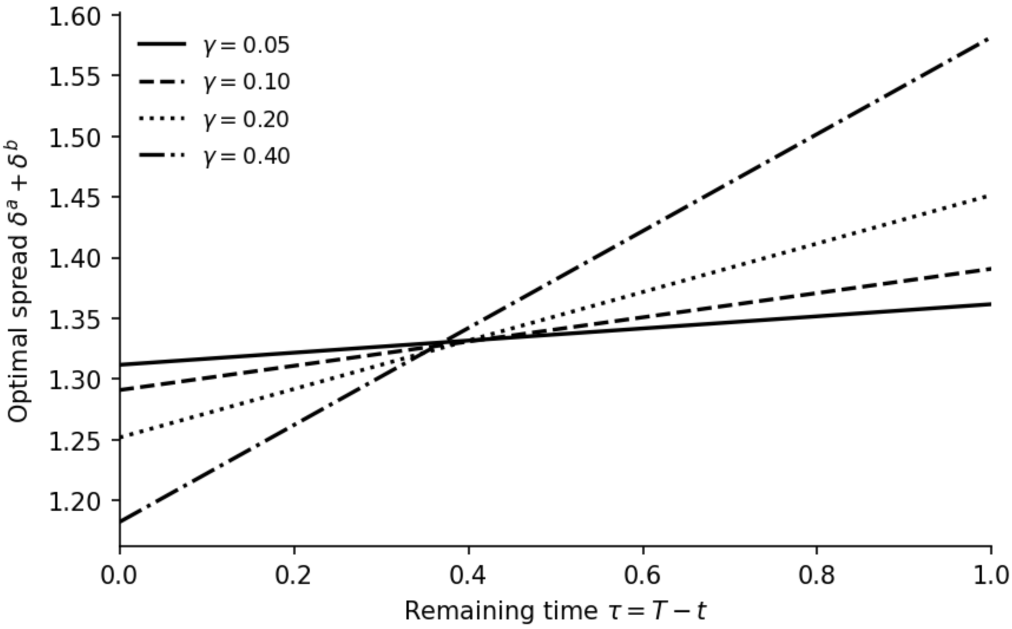

Figure 4: Total optimal spread $\delta^a + \delta^b$ as a function of remaining time τ for four risk-aversion levels $\gamma \in \{0.05, 0.10, 0.20, 0.40\}$. More risk-averse dealers post wider spreads at all horizons, and the spread widens as the horizon grows due to the accumulating inventory exposure.

8. Conclusion

The Avellaneda–Stoikov framework provides a tractable closed-form solution for the optimal market-making problem under inventory risk. The key structural results are: (i) the reservation price shifts linearly with inventory, tilting both quotes toward the unwinding direction; (ii) the optimal spread decomposes additively into a risk component and an adverse-selection component; (iii) the policy is self-stabilising — inventory deviations are corrected through asymmetric quoting without requiring explicit mean-reversion constraints.

Extensions include stochastic volatility in the mid-price, non-exponential order-arrival models (e.g. power-law tails), multiple assets with correlated inventory, and learning frameworks in which the dealer updates beliefs about the fraction of informed traders in real time.

9. References

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