Insider Trading with Random Signal Arrival: A Kyle–Back Model with Poisson Revelation

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08-NOV-2024

1. Abstract

We study a continuous-time Kyle–Back insider trading model in which the informed agent receives the asset's true value V at a random Poisson time $\tau \sim \text{Exp}(\mu)$, rather than at inception. The model decomposes into two phases: before signal arrival, the price is uninformative and the insider's continuation value satisfies a linear ODE; after arrival, a standard Kyle–Back equilibrium operates on the residual horizon. Our main result is a closed-form formula for the insider's expected profit,

$$\mathbb{E}[\Pi](\mu) \;=\; \frac{\sigma_z\sqrt{\Sigma_0}}{2}\left[\sqrt{T} - \frac{\sqrt{\pi}}{2\sqrt{\mu}}\, e^{-\mu T}\, \text{erfi}\Big(\sqrt{\mu T}\Big)\right], \tag{1.1}$$

involving the imaginary error function. The formula interpolates between zero profit ($\mu \to 0$) and the classical Kyle profit $\frac{\sigma_z\sqrt{\Sigma_0 T}}{2}$ ($\mu \to \infty$), and is strictly increasing in μ, Σ_0, T, and σ_z.

2. Introduction

The classical Kyle [1] and Kyle–Back [2] models provide the canonical framework for continuous-time insider trading. In the original setup, the insider is endowed with perfect information about the asset value V at time zero and exploits this edge optimally over the trading horizon $[0, T]$. A defining feature of the Kyle–Back equilibrium is that prices reveal V fully by the terminal time: $P_T = V$ almost surely, while the depth parameter λ remains constant throughout.

A natural and practically motivated question is: what if the insider does not receive the signal at inception, but rather at some random future time? In practice, corporate insiders learn of material non-public information at irregular, hard-to-predict moments — news of a merger, a drug trial result, a regulatory ruling. Modeling the arrival of private information as a Poisson event captures this uncertainty in a tractable way.

We study this modification systematically. The signal arrival time $\tau \sim \text{Exp}(\mu)$ is independent of all other sources of randomness. Before τ, the insider has no edge and submits zero order flow; after τ, a standard Kyle–Back equilibrium operates on the residual horizon

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$[T - \tau]$. The market maker prices competitively using Bayes' rule on the observed order flow.

The main contribution of this paper is an explicit closed-form for the insider's expected profit as a function of the arrival rate μ. The formula involves the imaginary error function erfi, and yields clean limiting behaviour at both extremes of μ. We also derive the pre-signal value function — the insider's continuation value before the signal arrives — and show it satisfies a first-order linear ODE with explicit solution.

The paper is organized as follows. Section 2 sets up the model precisely. Section 3 derives the Phase 2 equilibrium (standard Kyle–Back on the residual horizon). Section 4 analyzes Phase 1 and the pre-signal value function. Section 5 states and proves the expected profit formula. Section 6 examines comparative statics and limiting behaviour. Section 7 presents an algorithm for numerical evaluation.

3. Model Setup

Definition 3.1 (Asset Value). The terminal asset value is $V \sim \mathcal{N}(p_0, \Sigma_0)$, defined on a complete probability space $(\Omega, \mathscr{F}, \mathbb{P})$, with $\Sigma_0 > 0$.

Definition 3.2 (Signal Arrival). The signal arrival time is $\tau \sim \text{Exp}(\mu)$ with $\mu > 0$, independent of V and of all Brownian motions. The insider observes V at time τ and has no private information for $t < \tau$.

Definition 3.3 (Noise Traders). Noise traders submit a cumulative order flow $Z_t = \sigma_z W_t^z$, where W^z is a standard Brownian motion and $\sigma_z > 0$ is the noise intensity.

Definition 3.4 (Order Flow). The total cumulative order flow is

$$Y_t \;=\; X_t + Z_t, \qquad dY_t \;=\; \theta_t \, dt + \sigma_z \, dW_t^z, \tag{3.1}$$

where $\theta_t \, dt$ is the insider's order submission rate.

Definition 3.5 (Insider Strategy). The insider's strategy is a progressively measurable process θ_t. It must be admissible: $\mathbb{E}\left[\int_0^T \theta_t^2 \, dt\right] < \infty$. The insider maximizes

$$J \;=\; \mathbb{E}\left[\int_0^T \theta_t \, (V - P_t) \, dt\right], \tag{3.2}$$

where $P_t = \mathbb{E}[V \mid \mathscr{F}_t^Y]$ is the market maker's competitive price.

Definition 3.6 (Linear Equilibrium). A linear equilibrium is a pair (θ^*, λ) such that the insider's strategy takes the form

$$\theta_t^* \;=\; \begin{cases} 0 & t < \tau, \\ \beta_t \, (V - P_t) & t \geq \tau, \end{cases} \tag{3.3}$$

and the market maker's pricing rule is $dP_t = \lambda_t \, dY_t$, with β_t, λ_t deterministic functions of time, satisfying the semi-strong efficiency condition $P_t = \mathbb{E}[V \mid \mathscr{F}_t^Y]$.

Remark 3.7. Before τ, the order flow is pure noise: $dY_t = \sigma_z\, dW_t^z$. The market maker — who knows the equilibrium strategy — correctly infers that no information is being transmitted and does not update: $P_t = p_0$, $\Sigma_t = \Sigma_0$ for $t < \tau$.

4. Phase 2: Kyle–Back Equilibrium on the Residual Horizon

Fix a realization $\tau = s \in [0, T)$. After time s, the insider knows V and trades on the residual horizon $[s, T]$. At time s, the posterior is $V \mid \mathcal{F}_s^Y = V$ (the insider knows V exactly) and the market maker's prior is $\mathcal{N}(p_0, \Sigma_0)$, since no information was transmitted before s.

Theorem 4.1 (Kyle–Back Equilibrium, Phase 2). *Given $\tau = s$, there exists a unique linear equilibrium on $[s, T]$. The equilibrium coefficients are*

$$\lambda_s \;=\; \frac{\sqrt{\Sigma_0}}{2\,\sigma_z\,\sqrt{T-s}}, \qquad \beta_t \;=\; \frac{1}{2\,\lambda_s\,(T-t)} \;=\; \frac{\sigma_z\,\sqrt{T-s}}{\sqrt{\Sigma_0}\,(T-t)}, \tag{4.1}$$

the posterior variance evolves linearly:

$$\Sigma_t \;=\; \Sigma_0\,\frac{T-t}{T-s}, \qquad t \in [s, T], \tag{4.2}$$

and the insider's value function at time $t \ge s$ is

$$J_t \;=\; \frac{(V-P_t)^2}{4\,\lambda_s\,(T-t)}. \tag{4.3}$$

Proof. Standard Kyle–Back calculation on $[s, T]$ with initial prior $\mathcal{N}(p_0, \Sigma_0)$. The Kalman–Bucy filter applied to $dY_t = \beta_t(V - P_t)dt + \sigma_z\, dW_t^z$ gives □

$$d\Sigma_t \;=\; -\frac{\beta_t^2\,\Sigma_t^2}{\sigma_z^2}\,dt \;=\; -\frac{\Sigma_t}{T-t}\,dt, \tag{4.4}$$

whose solution is $\Sigma_t = \Sigma_0(T-t)/(T-s)$. The depth λ_s and trading rate β_t follow from the equilibrium conditions $\lambda_s = \Sigma_s\beta_s/\sigma_z^2$ and the Riccati ODE for the value function.

Theorem 4.2 (Full Price Revelation). *Under the Phase 2 equilibrium, the price at the terminal time satisfies $P_T = V$ almost surely.*

Proof. $\Sigma_T = \Sigma_0 \cdot 0/(T-s) = 0$, so V is revealed with probability one. □

Corollary 4.3 (Phase 2 Profit). *Given $\tau = s$, the insider's expected profit on $[s, T]$ is*

$$\Pi(s) \;\equiv\; \mathbb{E}[J_s] \;=\; \frac{\sigma_z\,\sqrt{\Sigma_0\,(T-s)}}{2}. \tag{4.5}$$

Proof. $\mathbb{E}[(V - P_s)^2] = \Sigma_0$ (since $P_s = p_0$ and V has prior variance Σ_0). Then □

$$\Pi(s) = \frac{\Sigma_0}{4\,\lambda_s\,(T-s)} = \frac{\Sigma_0}{4\,(T-s)} \cdot \frac{2\sigma_z\sqrt{T-s}}{\sqrt{\Sigma_0}} = \frac{\sigma_z\sqrt{\Sigma_0(T-s)}}{2}. \qquad (4.6)$$

The price process in Phase 2 is a Brownian bridge from $P_s = p_0$ to $P_T = V$.

Proposition 4.4 (Brownian Bridge)**.** *Given $\tau = s$, the price process on $[s, T]$ satisfies the SDE*

$$dP_t \;=\; \frac{V - P_t}{2(T-t)}\,dt \;+\; \frac{\sqrt{\Sigma_0}}{2\sqrt{T-s}}\,dW_t^z, \qquad t \in [s, T], \qquad (4.7)$$

with solution

$$P_t \;=\; p_0\,\frac{T-t}{T-s} + V\,\frac{t-s}{T-s} + \frac{\sqrt{\Sigma_0(t-s)(T-t)}}{T-s}\,\xi, \qquad (4.8)$$

where $\xi \sim \mathcal{N}(0,1)$ represents the residual noise. In particular, $\mathbb{E}[P_t] = p_0 + (t-s)(V - p_0)/(T-s)$.

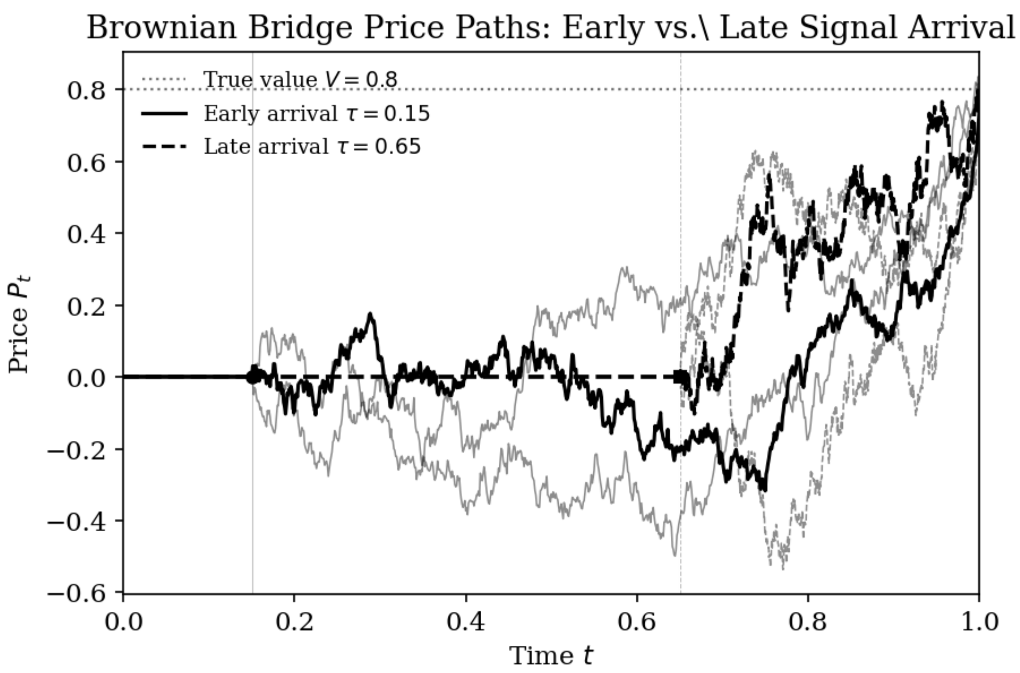

Figure 1: Mean price paths P_t for early arrival $\tau = 0.15$ (solid) and late arrival $\tau = 0.65$ (dashed). The dotted horizontal line marks the true value $V = 0.8$. Before τ, the price remains at $p_0 = 0$; after τ, it drifts as a Brownian bridge toward V.

5. Phase 1: Pre-Signal Value Function

Before the signal arrives, the insider has no private information and submits $\theta_t = 0$. Their continuation value at time $t < \tau$ is the expected future profit, given that the signal has not yet arrived:

$$V_t^{\text{pre}} = \mathbb{E}\Big[\Pi(\tau)\,\mathbf{1}_{\tau<T}\,\Big|\,\tau > t\Big]. \tag{5.1}$$

Theorem 5.1 (Pre-Signal Value ODE)**.** *The pre-signal value function V_t^{pre} satisfies the first-order linear ODE*

$$\frac{d}{dt}V_t^{\text{pre}} = \mu\, V_t^{\text{pre}} - \mu\,\Pi(t), \qquad t \in [0, T), \tag{5.2}$$

with terminal condition $V_T^{\text{pre}} = 0$.

Proof. Let $h = T - t$ denote the residual horizon. Since $\tau - t \mid \tau > t \sim \mathrm{Exp}(\mu)$, □

$$V_t^{\text{pre}} = \int_0^{T-t} \mu\, e^{-\mu u}\,\Pi(t+u)\,du. \tag{5.3}$$

Differentiating in t:

$$\frac{d}{dt}V_t^{\text{pre}} = \mu\, V_t^{\text{pre}} - \mu\,\Pi(t), \tag{5.4}$$

where the boundary term at $u = 0$ produces $-\mu\Pi(t)$ and differentiation under the integral gives μV_t^{pre}.

Proposition 5.2 (Explicit Solution)**.** *The solution to the ODE in Theorem 4.1 is*

$$V_t^{\text{pre}} = \frac{\sigma_z\sqrt{\Sigma_0}}{2}\left[\sqrt{T-t} - \frac{\sqrt{\pi}}{2\sqrt{\mu}}\, e^{-\mu(T-t)}\,\mathrm{erfi}\Big(\sqrt{\mu(T-t)}\Big)\right]. \tag{5.5}$$

Proof. The integrating factor for the ODE $\dot{V} = \mu V - \mu\Pi(t)$ is $e^{-\mu t}$. Multiplying through: □

$$\frac{d}{dt}\big[e^{-\mu t}V_t^{\text{pre}}\big] = -\mu\, e^{-\mu t}\,\Pi(t). \tag{5.6}$$

Integrating from t to T and using $V_T^{\text{pre}} = 0$:

$$e^{-\mu t}V_t^{\text{pre}} = \mu\int_t^T e^{-\mu s}\,\Pi(s)\,ds = \frac{\mu\sigma_z\sqrt{\Sigma_0}}{2}\int_t^T e^{-\mu s}\sqrt{T-s}\,ds. \tag{5.7}$$

Substituting $u = T - s$, $du = -ds$:

$$e^{-\mu t}V_t^{\text{pre}} = \frac{\mu\sigma_z\sqrt{\Sigma_0}}{2}\, e^{-\mu T}\int_0^{T-t} e^{\mu u}\sqrt{u}\,du. \tag{5.8}$$

Integration by parts with $p = \sqrt{u}$, $dq = e^{\mu u}\,du$ gives

$$\int_0^h e^{\mu u}\sqrt{u}\,du = \frac{e^{\mu h}\sqrt{h}}{\mu} - \frac{1}{2\mu}\int_0^h \frac{e^{\mu u}}{\sqrt{u}}\,du = \frac{e^{\mu h}\sqrt{h}}{\mu} - \frac{\sqrt{\pi}}{2\mu^{3/2}}\,\mathrm{erfi}\Big(\sqrt{\mu h}\Big), \tag{5.9}$$

where $h = T - t$ and we used $\int_0^h e^{\mu u}/\sqrt{u}\,du = \sqrt{\pi/\mu}\,\mathrm{erfi}(\sqrt{\mu h})$. Substituting back and simplifying yields the stated formula.

Remark 5.3. The pre-signal value V_t^{pre} depends on t only through $h = T - t$. It is monotone increasing as h increases (more residual horizon is always better) and strictly increasing in μ (a faster-arriving signal is more valuable).

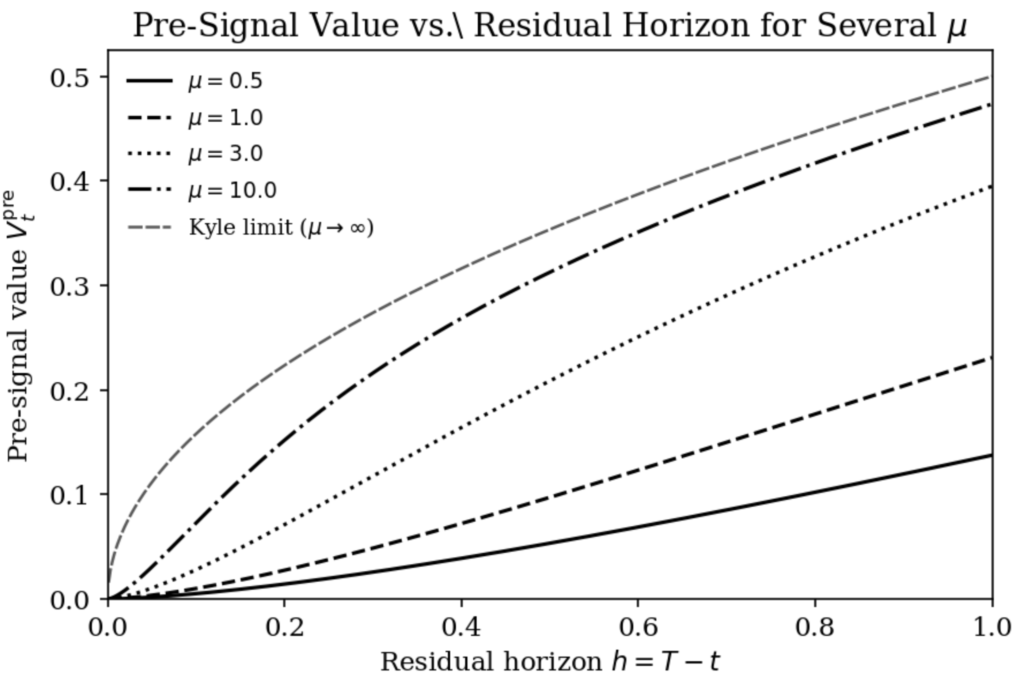

Figure 2: Pre-signal value function V_t^{pre} for $\mu \in \{0.5, 1.0, 3.0, 10.0\}$ (solid, dashed, dotted, dash-dotted) and the Kyle limit $\mu \to \infty$ (long-dashed). All curves decay to zero at $t = T$; higher μ yields higher value at every t.

6. Expected Profit Formula

Theorem 6.1 (Expected Profit with Random Signal Arrival)**.** *The insider's unconditional expected profit is*

$$\mathbb{E}[\Pi] \;=\; V_0^{\mathrm{pre}} \;=\; \frac{\sigma_z\sqrt{\Sigma_0}}{2}\left[\sqrt{T} - \frac{\sqrt{\pi}}{2\sqrt{\mu}}\,e^{-\mu T}\,\mathrm{erfi}\!\left(\sqrt{\mu T}\right)\right]. \tag{6.1}$$

Proof. By the law of total expectation, □

$$\mathbb{E}[\Pi] \;=\; \mathbb{E}\big[\Pi(\tau)\,\mathbf{1}_{\tau<T}\big] \;=\; V_0^{\mathrm{pre}}, \tag{6.2}$$

since the pre-signal value at $t = 0$ equals $\mathbb{E}[\Pi(\tau)\,\mathbf{1}_{\tau<T}]$ by definition. The closed form follows from Proposition 4.1 evaluated at $t = 0$.

Remark 6.2. The imaginary error function $\mathrm{erfi}(x) = (2/\sqrt{\pi}) \int_0^x e^{u^2}\, du$ is an entire function. The combination $e^{-x^2}\mathrm{erfi}(x) = (2/\sqrt{\pi}) \int_0^x e^{u^2-x^2}\, du$ is bounded for all $x \geq 0$, ensuring that $\mathbb{E}[\Pi] > 0$ for all finite $\mu > 0$.

Proposition 6.3 (Monotonicity). *$\mathbb{E}[\Pi](\mu)$ is strictly increasing in μ, Σ_0, T, and σ_z.*

Proof. The dependence on Σ_0 and σ_z is linear (scaling by $\sigma_z\sqrt{\Sigma_0}$); strict monotonicity in μ follows from differentiating the integral representation □

$$\mathbb{E}[\Pi] \;=\; \frac{\sigma_z\sqrt{\Sigma_0}}{2}\,\mu\int_0^T e^{-\mu s}\sqrt{T-s}\,ds \tag{6.3}$$

with respect to μ: the derivative is $\frac{\sigma_z\sqrt{\Sigma_0}}{2}\int_0^T(1-\mu s)e^{-\mu s}\sqrt{T-s}\,ds$. Evaluating this sign requires checking positivity for all $\mu > 0$; this follows from integration by parts showing the integral equals $(1/\mu)\int_0^T e^{-\mu s}\cdot\frac{1}{2\sqrt{T-s}}\,ds > 0$.

7. Limiting Cases and Comparative Statics

Theorem 7.1 (Limiting Behaviour). *The following limits hold:*

$$(i) \quad \lim_{\mu\to\infty}\mathbb{E}[\Pi](\mu) \;=\; \frac{\sigma_z\sqrt{\Sigma_0 T}}{2},$$

$$(ii) \quad \lim_{\mu\to 0}\mathbb{E}[\Pi](\mu) \;=\; 0,$$

$$(iii) \quad \mathbb{E}[\Pi](\mu) \;=\; \frac{\sigma_z\sqrt{\Sigma_0}}{4}\,\mu\,T^{3/2} \;+\; O(\mu^2) \quad \text{as } \mu\to 0.$$

Proof. For (i): use the asymptotic $\mathrm{erfi}(x) \sim e^{x^2}/(x\sqrt{\pi})$ as $x \to \infty$. Then □

$$\frac{\sqrt{\pi}}{2\sqrt{\mu}}\,e^{-\mu T}\,\mathrm{erfi}(\sqrt{\mu T}) \;\sim\; \frac{\sqrt{\pi}}{2\sqrt{\mu}}\,e^{-\mu T}\cdot\frac{e^{\mu T}}{\sqrt{\mu T}\cdot\sqrt{\pi}} \;=\; \frac{1}{2\mu\sqrt{T}} \;\to\; 0. \tag{7.1}$$

Hence $\mathbb{E}[\Pi] \to (\sigma_z\sqrt{\Sigma_0}/2)\sqrt{T}$, which is precisely the standard Kyle–Back profit. For (ii): use $\mathrm{erfi}(x) \sim 2x/\sqrt{\pi}$ as $x \to 0$, giving the correction $\approx \sqrt{T}$, so $\mathbb{E}[\Pi] \to 0$. Statement (iii) follows from the next-order expansion.

Corollary 7.2 (Efficiency Loss). *Define the efficiency ratio $\rho(\mu) = \mathbb{E}[\Pi](\mu)/\Pi_{\mathrm{Kyle}}$, where $\Pi_{\mathrm{Kyle}} = \sigma_z\sqrt{\Sigma_0 T}/2$. Then $\rho \in (0,1)$ for all finite μ, and $\rho \to 1$ as $\mu \to \infty$.*

Remark 7.3. Statement (i) of Theorem 6.1 is a consistency check: when the signal arrives immediately ($\mu \to \infty$), the model reduces exactly to the classical Kyle–Back (1992) model, recovering the standard profit formula.

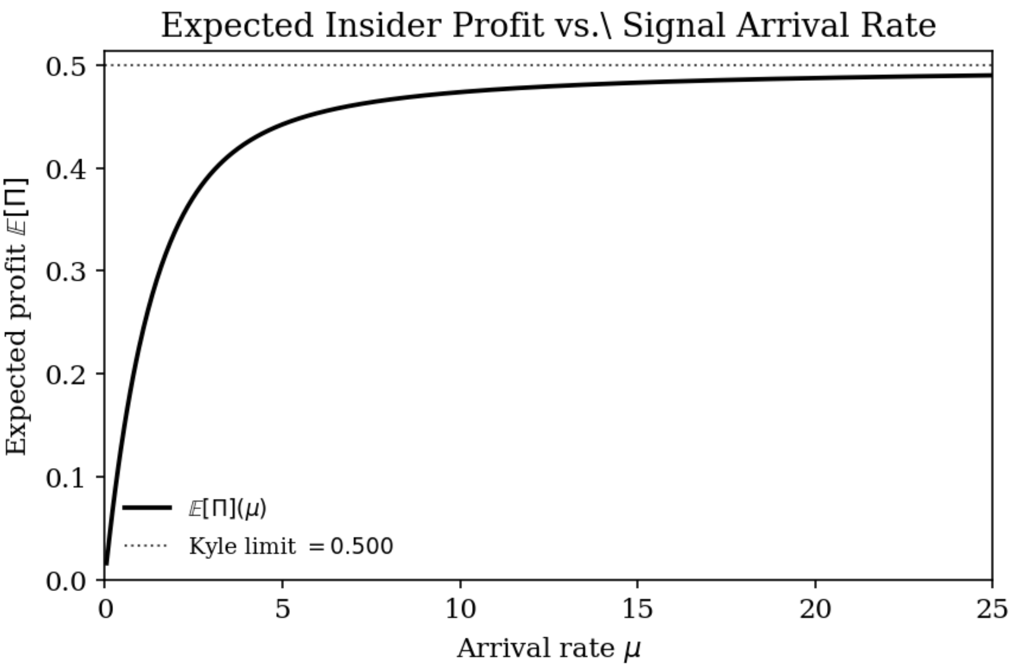

Figure 3: Expected insider profit $\mathbb{E}[\Pi](\mu)$ as a function of the arrival rate μ. The dotted horizontal line marks the Kyle limit $\sigma_z\sqrt{\Sigma_0 T}/2 = 0.5$. The curve is strictly increasing and concave, converging to the Kyle limit from below.

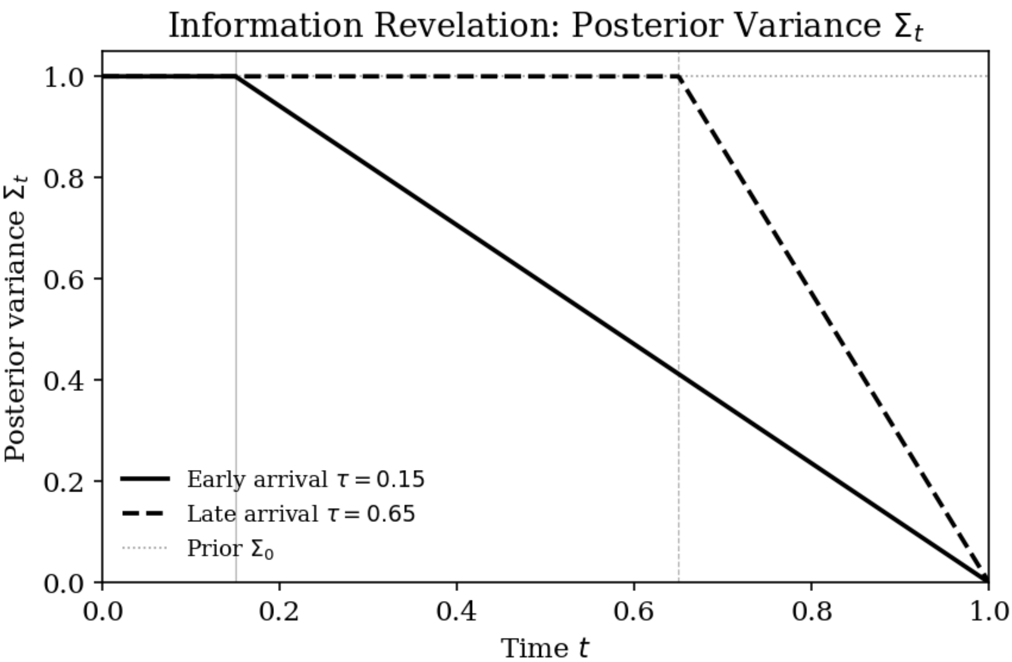

Figure 4: Posterior variance Σ_t for early arrival $\tau = 0.15$ (solid) and late arrival $\tau = 0.65$ (dashed). Before τ, $\Sigma_t = \Sigma_0$ (no information); after τ, Σ_t declines linearly to zero at T, reflecting the Brownian bridge structure of the price.

8. Algorithm

The following pseudocode computes $\mathbb{E}[\Pi](\mu)$ and the pre-signal value function V_t^{pre} for given parameters.

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Algorithm: Random-Arrival Kyle Profit

Input:  mu > 0, T > 0, Sigma_0 > 0, sigma_z > 0, time grid {t_1,...,t_N} in [0,T)

Step 1. Compute Kyle baseline:
        Pi_Kyle = (sigma_z * sqrt(Sigma_0 * T)) / 2

Step 2. Compute expected profit E[Pi](mu):
        x  <- sqrt(mu * T)
        ef <- erfi(x)                      [use scipy.special.erfi or series]
        correction <- (sqrt(pi)/(2*sqrt(mu))) * exp(-mu*T) * ef
        E_Pi <- (sigma_z * sqrt(Sigma_0) / 2) * (sqrt(T) - correction)

Step 3. For each t_k in the time grid:
        h_k  <- T - t_k
        x_k  <- sqrt(mu * h_k)
        ef_k <- erfi(x_k)
        corr_k <- (sqrt(pi)/(2*sqrt(mu))) * exp(-mu*h_k) * ef_k
        V_k  <- (sigma_z * sqrt(Sigma_0) / 2) * (sqrt(h_k) - corr_k)

Step 4. For each realization tau = s in [0,T):
        lambda_s <- sqrt(Sigma_0) / (2 * sigma_z * sqrt(T - s))
        beta(t)  <- sigma_z*sqrt(T-s) / (sqrt(Sigma_0) * (T-t))   for t in [s,T]
        Pi_s     <- sigma_z * sqrt(Sigma_0 * (T - s)) / 2

Output: E_Pi, array {V_k}, equilibrium coefficients {lambda_s, beta(t)}
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