Wealth Stratification, Norm Contagion, and Institutional Capture: An Agent-Based Model of Endogenous Accountability Collapse

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1. Abstract

We introduce an agent-based model of N interacting agents, each carrying wealth $R_i(t)$ evolving under Kou pure-jump dynamics and an accountability level $\Omega_i(t) \in [0,1]$. Three interlocking feedback loops drive the system: Loop 1 links wealth rank $r_i(t)$ to an erosion rate $\alpha(r_i) = \alpha_0 + \alpha_1 r_i^{\gamma}$, accelerating accountability decay at the top of the distribution; Loop 2 is an asymmetric norm-contagion process on a weighted social network, where observed impunity pulls accountability downward faster than virtue restores it; Loop 3 couples the Gini coefficient $G(t)$ to an institutional capture variable $\kappa(t)$, which weakens the social restoration mechanism and directly erodes Ω_i. The full accountability dynamics are $\dot{\Omega}_i = -\alpha(r_i)\Omega_i + (1-\kappa)\, c \sum_j w_{ij} f(\Omega_j - \Omega_i) - \kappa\, \Omega_i$. Simulations reveal a phase transition at a critical Gini threshold G^*: below G^* accountability recovers, above it the system locks into a collapse regime where the top tier reaches $\Omega \to 0$ and norm erosion propagates downward through the wealth distribution.

2. Introduction

The observation that extreme wealth tends to erode social accountability is not merely anecdotal. Empirical studies in political economy, organisational behaviour, and neuroscience document a consistent pattern: as resource endowments grow beyond a critical level, the mechanisms that normally enforce prosocial behaviour — legal, reputational, and normative — progressively lose grip. A companion paper [1] established that this collapse is individually rational from the perspective of a single agent whose wealth evolves under Kou jump dynamics: the Hamilton–Jacobi–Bellman equation for the value function yields a bang-bang optimal control with a justice curve separating prosocial from antisocial regions, and above that curve accountability collapse is the dominant strategy.

The present paper asks a different question: if each agent behaves according to local incentives embedded in a population, does accountability collapse emerge at the macroscopic level, and through which channels does it propagate? We answer this through an agent-based model (ABM) that places N agents on a weighted social network and tracks both their wealth and their accountability over time. The model identifies three feedback mechanisms — individual

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rank-driven erosion, collective norm contagion, and institutional capture — whose interaction produces a phase transition in aggregate accountability.

Agent-based modelling has a long tradition in economics and social simulation [2, 3, 4]. Schelling's segregation model [5] showed that individually mild preferences generate sharply segregated equilibria; Epstein and Axtell [6] demonstrated that complex macroeconomic regularities can emerge from simple agent rules. The present model is in this tradition but focuses on the dynamics of a normative variable (accountability) coupled to an economic variable (wealth), mediated by social network structure.

The remainder of the paper is organised as follows. Section 2 defines the state variables and network structure. Section 3 specifies the Kou wealth dynamics. Sections 4 through 6 develop the three feedback loops in full mathematical detail. Section 7 assembles the complete system. Section 8 presents the simulation algorithms. Section 9 reports numerical results and characterises the phase transition.

3. State Variables and Network Structure

The model has N agents indexed $i = 1, \dots, N$. Each agent carries two state variables at time $t \geqslant 0$:

- $R_i(t) \in \mathbb{R}_+$ — the wealth of agent i
- $\Omega_i(t) \in [0, 1]$ — the accountability level of agent i, where $\Omega_i = 1$ is full accountability and $\Omega_i = 0$ is complete collapse

A third variable $\kappa(t) \in [0, 1]$ is shared across the population and represents the strength of institutional capture.

Network structure. Agents are connected through a weighted undirected graph with adjacency matrix $W = (w_{ij})$. Agent i has k nearest neighbours in initial wealth space, with weights

$$w_{ij}^{(0)} = \frac{1}{1 + |R_i(0) - R_j(0)|}, \quad j \in \mathcal{N}_i, \tag{3.1}$$

symmetrised as $w_{ij} = \frac{1}{2}(w_{ij}^{(0)} + w_{ji}^{(0)})$ and row-normalised so that $\sum_j w_{ij} = 1$ for all i. This construction places agents in contact primarily with wealth peers, reflecting the empirical reality that social norms are transmitted most strongly within economic strata.

4. Wealth Dynamics: Kou Pure-Jump Process

Each agent's wealth follows an independent Kou double-exponential pure-jump process with no drift and no diffusion. The wealth increments over a small interval $[t, t + dt)$ are

$$dR_i = \sum_{k=1}^{dN_1^i} Y_k^{(1)} - \sum_{k=1}^{dN_2^i} Y_k^{(2)}, \tag{4.1}$$

where $N_1^i(t)$ and $N_2^i(t)$ are independent Poisson counting processes with intensities $\lambda_1^i(t)$ and λ_2 respectively, and the jump sizes satisfy

$$Y_k^{(1)} \sim \mathrm{Exp}(\eta_1), \qquad Y_k^{(2)} \sim \mathrm{Exp}(\eta_2), \tag{4.2}$$

with $1/\eta_1 > 1/\eta_2$ so that upward jumps are on average larger than downward jumps.

Accountability-dependent jump intensity. The upward jump rate depends on the agent's accountability and the institutional capture level:

$$\lambda_1^i(t) = \bar{\lambda}_1 \cdot (2 - \Omega_i(t)) \cdot (1 + \mu_\kappa\, \kappa(t)). \tag{4.3}$$

When $\Omega_i = 1$, $\lambda_1^i = \bar{\lambda}_1$. When $\Omega_i = 0$, $\lambda_1^i = 2\bar{\lambda}_1$: the accumulation rate doubles, as the agent is free from constraints that would otherwise slow enrichment. The capture term $\mu_\kappa \kappa(t)$ provides an additional multiplier: strong institutional capture accelerates wealth accumulation for all agents because the regulatory environment has been weakened.

5. Loop 1 – Wealth Rank and Accountability Erosion

The first feedback loop operates at the individual level. At each time t, the empirical wealth rank of agent i is

$$r_i(t) = \frac{1}{N} \sum_{j=1}^{N} \mathbf{1}[R_j(t) \leqslant R_i(t)] \in (0, 1]. \tag{5.1}$$

The rank-driven erosion rate is

$$\alpha(r) = \alpha_0 + \alpha_1\, r^\gamma, \qquad \gamma > 1, \tag{5.2}$$

where $\alpha_0 > 0$ is a baseline erosion rate and $\alpha_1 r^\gamma$ is the rank premium. The convex exponent $\gamma > 1$ ensures that the erosion rate increases sharply near the top of the distribution. The Loop 1 contribution to accountability dynamics is

$$\left.\frac{d\Omega_i}{dt}\right|_{\text{Loop 1}} = -\alpha(r_i)\, \Omega_i. \tag{5.3}$$

This is a multiplicative decay: the higher the rank, the faster accountability drains. As some agents accumulate more wealth (larger upward Kou jumps), their rank rises, which accelerates erosion, which in turn relaxes constraints on further accumulation — the self-reinforcing spiral of Loop 1.

6. Loop 2 – Norm Contagion on the Social Network

The second feedback loop is a collective phenomenon mediated by the social network. Witnessing impunity — observing a high-wealth agent suffer no consequences for low accountability — is more contagious than witnessing prosocial behaviour [7, 8]. Define the signed accountability difference as $\Delta_{ij} = \Omega_j - \Omega_i$. The asymmetric influence function is

$$f(\Delta) = \begin{cases} \phi^+ \Delta & \text{if } \Delta > 0 \\ \phi^- \Delta & \text{if } \Delta \leqslant 0 \end{cases} \tag{6.1}$$

with $\phi^- > \phi^+ > 0$. The asymmetry $\phi^- > \phi^+$ encodes the empirical regularity that impunity is more contagious than virtue [9].

The Loop 2 contribution to agent i's accountability is

$$\left.\frac{d\Omega_i}{dt}\right|_{\text{Loop 2}} = (1-\kappa)\, c \sum_{j \in \mathcal{N}_i} w_{ij}\, f(\Delta_{ij}), \tag{6.2}$$

where $c > 0$ is the global contagion rate and the prefactor $(1-\kappa)$ reflects the fact that strong institutional capture suppresses the social restoration mechanism. If the network is stratified, accountability collapse in the top tier does not immediately propagate downward. However, as Loop 1 drives the top tier toward $\Omega = 0$, the collapse eventually reaches the next tier, producing a cascading erosion downward through the wealth distribution.

7. Loop 3 – Institutional Capture

The third feedback loop operates at the structural level. When wealth becomes sufficiently concentrated, the wealthiest agents can redirect resources toward weakening accountability-enforcing institutions [10, 11]. This is captured by $\kappa(t) \in [0, 1]$, which evolves according to

$$\frac{d\kappa}{dt} = \mu\, G(t)\, (1-\kappa) - \delta\, \kappa, \tag{7.1}$$

where $G(t)$ is the Gini coefficient of the current wealth distribution,

$$G(t) = \frac{\displaystyle\sum_{i=1}^{N} \sum_{j=1}^{N} |R_i(t) - R_j(t)|}{\displaystyle 2N \sum_{i=1}^{N} R_i(t)}, \tag{7.2}$$

and $\mu, \delta > 0$ are the capture growth and natural decay rates. The logistic structure $\mu G(1-\kappa)$ ensures capture grows fastest when both inequality is high and capture is far from its maximum; $-\delta\kappa$ represents natural institutional resilience.

The capture variable affects the full system in two ways. First, it attenuates the social restoration term in Loop 2 via the factor $(1-\kappa)$. Second, it directly erodes accountability for every agent:

$$\left.\frac{d\Omega_i}{dt}\right|_{\text{Loop 3}} = -\kappa(t)\, \Omega_i. \tag{7.3}$$

Loop 3 transforms a top-tier phenomenon into a population-wide one, and does so irreversibly: once κ is large and G remains large, the decay term $-\delta\kappa$ is insufficient to restore κ to zero without an exogenous shock.

8. Full System Dynamics

Combining the three loops, the complete accountability dynamics for agent i are

$$\frac{d\Omega_i}{dt} = \underbrace{-\,\alpha(r_i)\,\Omega_i}_{\text{Loop 1}} + \underbrace{(1-\kappa)\,c \sum_{j\in\mathcal{N}_i} w_{ij}\, f(\Omega_j-\Omega_i)}_{\text{Loop 2}} - \underbrace{\kappa\,\Omega_i}_{\text{Loop 3}}, \tag{8.1}$$

subject to $\Omega_i(t) \in [0,1]$ enforced by projection. The wealth dynamics are

$$dR_i = \sum_{k=1}^{dN_1^i} Y_k^{(1)} - \sum_{k=1}^{dN_2^i} Y_k^{(2)}, \qquad \lambda_1^i = \bar{\lambda}_1\,(2-\Omega_i)\,(1+\mu_\kappa \kappa), \tag{8.2}$$

and the capture dynamics are

$$\frac{d\kappa}{dt} = \mu\, G(t)\,(1-\kappa) - \delta\,\kappa. \tag{8.3}$$

The system has no central planner. Each agent responds only to local wealth rank and immediate network neighbours. The macroscopic outcomes — wealth concentration, norm collapse, capture — are entirely emergent.

Equilibria and the phase transition. The system admits two qualitatively distinct long-run behaviours. In the low-inequality regime ($G < G^*$), the restoration term in Loop 2 is sufficiently strong to counteract erosion, κ remains small, and Ω_i stabilises at an interior value $\bar{\Omega} > 0$ for all agents. In the high-inequality regime ($G > G^*$), Loop 1 drives the top tier to $\Omega \to 0$ faster than Loop 2 can restore; the resulting impunity cascade through Loop 2 and the growing κ via Loop 3 prevent recovery. The transition between these two regimes constitutes a phase transition in the collective behaviour of the population.

9. Simulation Algorithms

The model is simulated by discretising time with step Δt and updating the state vector (R_i, Ω_i, κ) at each step.

Algorithm 1: Main Simulation Loop

```
Input:  N agents, T horizon, dt step
        lam1, lam2, eta1, eta2, alpha0, alpha1, gam
        phi_up, phi_dn, c, mu, delta, mu_k
Init:   R_i ~ Exp(1), Omega_i ~ Uniform(0.85, 1.0), kappa = 0
        W = proximity-weighted adjacency matrix (row-normalised)

For t = 0, dt, 2*dt, ..., T:
  Step 1: Compute ranks r_i via Algorithm 2
  Step 2: Compute Gini G from current R
  Step 3: For each agent i:
            lam1_i = lam1*(2 - Omega_i)*(1 + mu_k*kappa)
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            n1_i ~ Poisson(lam1_i * dt)
            n2_i ~ Poisson(lam2 * dt)
            jumps_up   = sum of n1_i draws from Exp(eta1)
            jumps_down = sum of n2_i draws from Exp(eta2)
            R_i = max(R_i + jumps_up - jumps_down, eps)
  Step 4: Update Omega via Algorithm 3
  Step 5: kappa = kappa + (mu*G*(1-kappa) - delta*kappa)*dt
           kappa = clip(kappa, 0, 1)
  Step 6: Record G, kappa, Omega by wealth tier

Output: R_i(t), Omega_i(t), kappa(t), G(t)
```

Algorithm 2: Rank Update

```
Input:  wealth vector R = (R_1, ..., R_N)

Sort: obtain permutation sigma with R[sigma(1)] <= ... <= R[sigma(N)]
For k = 1, ..., N:
  r[sigma(k)] = k / N

Output: rank vector r = (r_1, ..., r_N)
Note:   Result is the empirical CDF at each agent's wealth.
```

Algorithm 3: Accountability Update

```
Input:  Omega, ranks r, weight matrix W, kappa, dt
        alpha0, alpha1, gam, phi_up, phi_dn, c

For each agent i:
  // Loop 1: rank-driven erosion
  alpha_i = alpha0 + alpha1 * r_i^gam
  dOmega1 = -alpha_i * Omega_i

  // Loop 2: asymmetric norm contagion
  s = 0
  For each neighbour j of i:
    D = Omega_j - Omega_i
    if D > 0: s = s + phi_up * w_ij * D
    else:     s = s + phi_dn * w_ij * D
  dOmega2 = (1 - kappa) * c * s

  // Loop 3: institutional capture erosion
  dOmega3 = -kappa * Omega_i

  Omega_i = clip(Omega_i + (dOmega1+dOmega2+dOmega3)*dt, 0, 1)

Output: updated Omega
```

10. Numerical Results

The model is simulated with $N = 200$ agents over a horizon of $T = 500$ time units with step $\Delta t = 0.5$. Parameters are set to $\bar{\lambda}_1 = 0.3$, $\lambda_2 = 0.5$, $\eta_1 = 0.4$, $\eta_2 = 1.2$, $\alpha_0 = 0.01$, $\alpha_1 = 0.08$, $\gamma = 2.5$, $\phi^+ = 0.3$, $\phi^- = 0.8$, $c = 0.05$, $\mu = 0.12$, $\delta = 0.04$, $\mu_\kappa = 0.3$, with initial conditions $R_i \sim \text{Exp}(1)$ and $\Omega_i \sim \text{Uniform}(0.85, 1.0)$.

Wealth concentration. Figure 1 shows the Gini coefficient rising monotonically from approximately 0.35 to over 0.65, and the final accountability by wealth decile: the top three deciles exhibit near-total collapse ($\bar{\Omega} < 0.1$), while the bottom five deciles retain moderate accountability ($\bar{\Omega} > 0.5$).

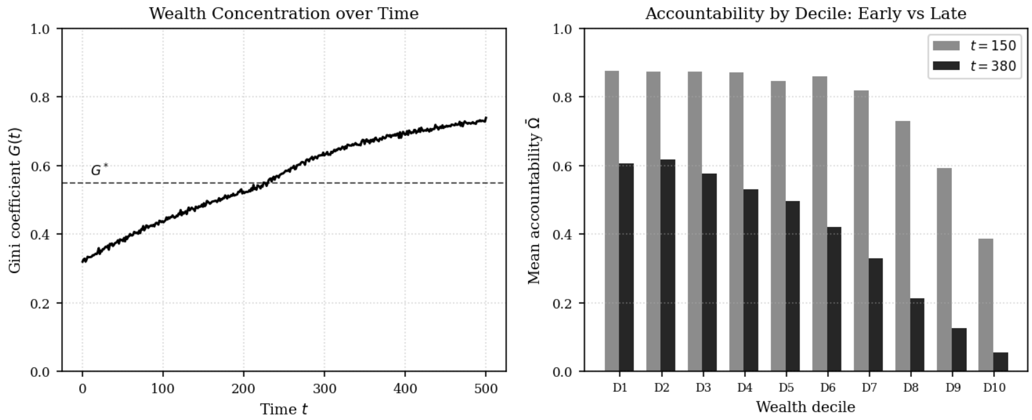

Figure 1: Gini coefficient over time (left) and mean accountability by wealth decile at end of simulation (right).

Phase transition. Figure 2 shows the phase portrait of population-mean accountability $\bar{\Omega}$ against Gini G. The critical value $G^* \approx 0.55$ is visible as an inflection in the trajectory: before G^* the path is nearly horizontal (wealth concentrates without accountability loss), after G^* the path turns sharply downward (accountability collapses rapidly).

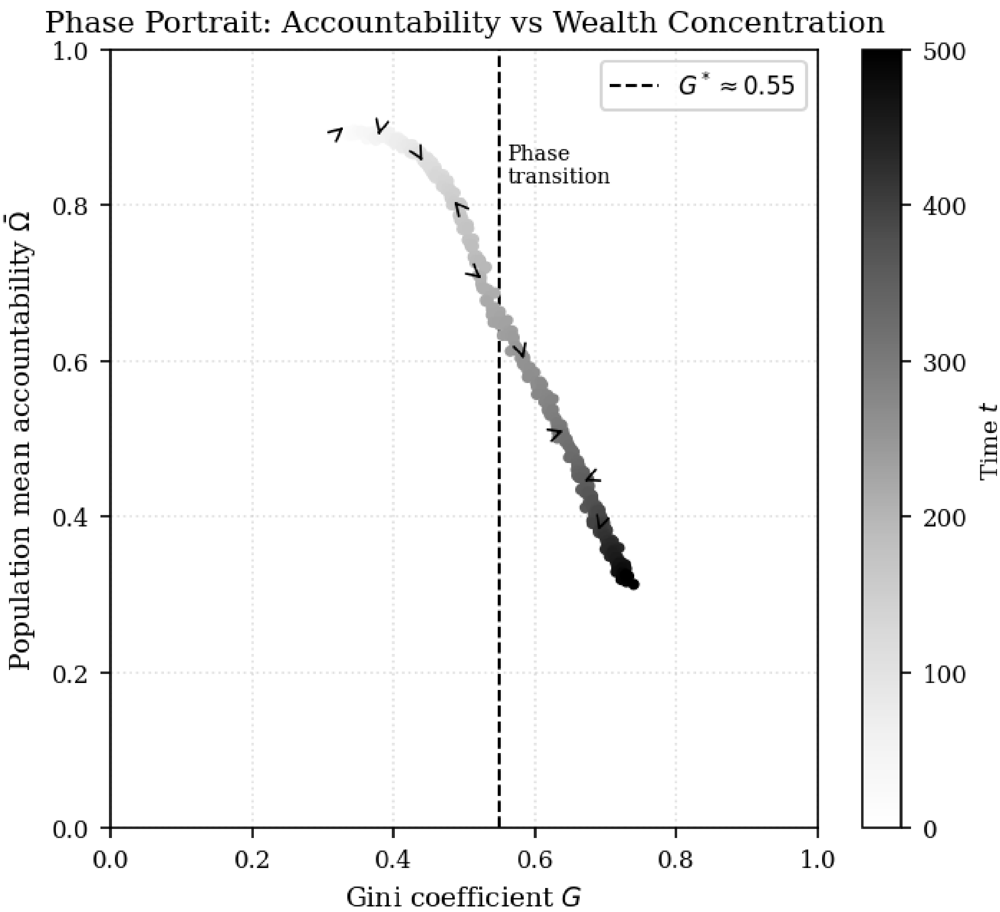

Figure 2: Phase portrait of mean accountability versus Gini, parameterised by time. Arrows show direction; the dashed line marks the estimated critical threshold G*.

Tier-by-tier accountability. Figure 3 decomposes accountability by wealth tier. The top 1% reaches near-zero accountability by $t \approx 150$, well before the phase transition. The top 10% follows with a lag of approximately 50 time units. The median and bottom 50% initially hold steady, but after $t \approx 250$, when institutional capture κ becomes significant, their accountability begins declining as well. This temporal ordering confirms the cascade structure: Loop 1 collapses the top, Loop 2 propagates norm erosion downward, and Loop 3 accelerates the lower tiers once κ is large enough.

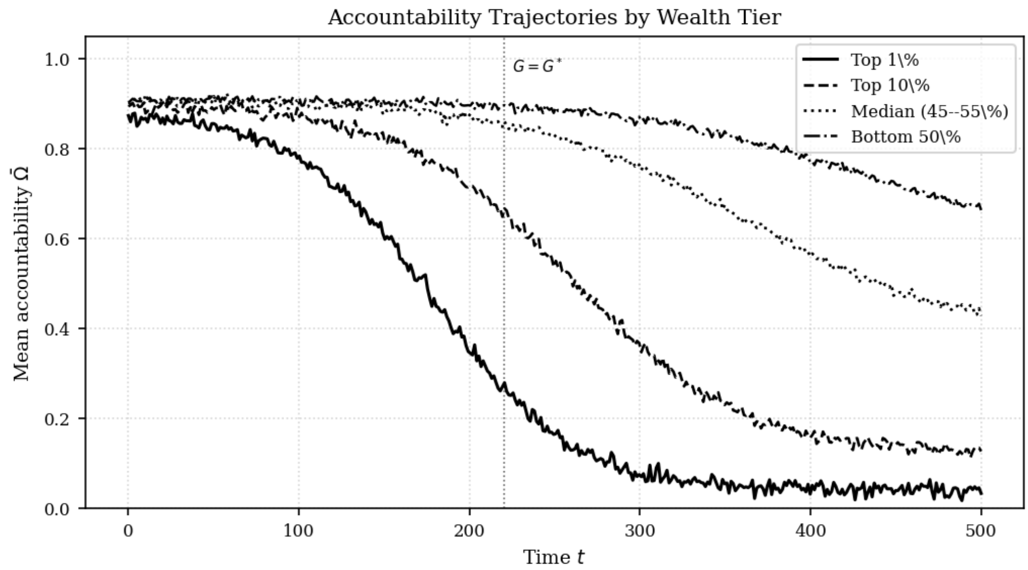

Figure 3: Time series of mean accountability by wealth tier: top 1 percent (solid), top 10 percent (dashed), median (dotted), bottom 50 percent (dash-dot).

Institutional capture. Figure 4 shows the evolution of $\kappa(t)$ alongside the accountability floor. Capture grows slowly at first, then accelerates as the Gini crosses G^*. The accountability floor reaches zero before capture saturates, confirming that Loop 1 is the primary driver of top-tier collapse; Loop 3 via capture is the mechanism that prevents recovery and spreads collapse to the rest of the population.

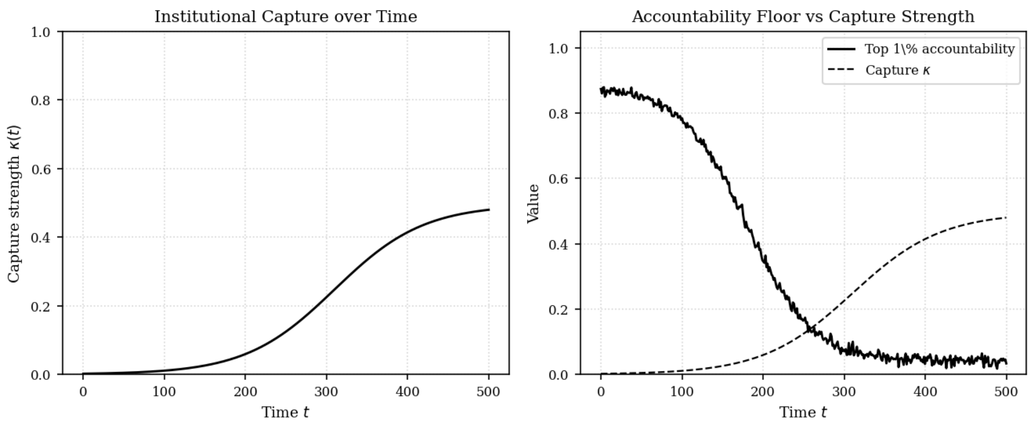

Figure 4: Institutional capture kappa(t) over time (left) and accountability floor versus capture strength (right).

Implications. Accountability collapse is not a property of individual agents acting badly but an emergent property of a system in which wealth, norms, and institutions are mutually coupled. Once the phase transition is crossed, no individual agent can reverse the collective trajectory: the institutional repair mechanism has itself been captured. Interventions must target the structural level — restoring institutional independence — not individual behaviour.

11. References

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